

Interrogation sur les opérateurs différentiels

$$1) \quad \text{div } \vec{A} = \begin{vmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{vmatrix} \cdot \begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{\text{rot}} \vec{A} = \begin{vmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{vmatrix} \wedge \begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix} = \begin{vmatrix} \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \\ \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \\ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \end{vmatrix}$$

$$2) \quad \vec{\text{rot}}(\vec{\text{grad}} f) = \vec{0} \quad \text{et} \quad \text{div}(\vec{\text{grad}} f) = \Delta f$$

$$3) \quad \text{div } \vec{v} = 0 \quad \text{et} \quad \vec{\text{rot}} \vec{v} = 2k \vec{e}_z$$

$$4) \quad (\vec{v} \cdot \vec{\text{grad}}) \vec{v} = \begin{vmatrix} \vec{v} \cdot \vec{\text{grad}} v_x \\ 0 \\ \vec{v} \cdot \vec{\text{grad}} v_z \end{vmatrix} = \begin{vmatrix} v_x \frac{\partial v_x}{\partial x} + v_z \frac{\partial v_x}{\partial z} \\ 0 \\ v_x \frac{\partial v_z}{\partial x} + v_z \frac{\partial v_z}{\partial z} \end{vmatrix}$$

$$= \begin{vmatrix} 2k^2 x^3 \\ 0 \\ 4y^2 z k^2 \end{vmatrix}$$